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# Factorization and generalized $*$-products

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**ABSTRACT**

The generalized $*$-products, or the $*_N$-products, appear both in the one-loop effective action of noncommutative Yang-Mills theories and in the coupling of a closed string to $N$ open strings on a disk when the D-brane world-volume is noncommutative. Factorization of the string amplitudes provides a uniform understanding of the $*_N$-products and a hint to obtain a simple, explicit formula (in the momentum space) for arbitrary $N$ in the non-Abelian case. Possible extension to a more general ${}_M{*}_N$-product in the $M$-loop context is discussed.

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Field theories on noncommutative $\mathbb{R}^n$ (NCFT) [1, 2] can be obtained by replacing the ordinary product between two fields with the $*$-product defined as

$$\phi_1 * \phi_2(x) = \exp\left[\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial z^\nu}\right]\phi_1(y)\phi_2(z)|_{y=z=x} \ . \tag{1}$$

By taking an appropriate decoupling limit where the string length scale goes to zero, the noncommutative field theories are obtained from string theory with a constant background NS-NS two-form field [3, 4]. In particular, the $*$-product in (1) is reproduced from the tree-level vertex operator product [4]

$$\exp\left(ip_1 X(x)\right)\exp\left(ip_2 X(x')\right) \simeq \exp\left(-p_{1\mu}\langle X^\mu(x)X^\nu(x')\rangle p_{2\nu}\right) \ , \tag{2}$$

where we suppressed the polarization dependent part of the vertex operators, with

$$\langle X^\mu(x)X^\nu(x')\rangle \simeq \frac{i}{2}\theta^{\mu\nu}\epsilon(x-x') + \cdots \ . \tag{3}$$

The function $\epsilon(x-x')$ is the Heaviside step function. In (2), the part of the world-sheet propagator that depends only on the "ordering" of the vertex operators is retained. Since every vertex operator in any string theory contains the piece shown in (3), the appearance of the $*$-product structure is quite generic and universal.

At the loop level[1], NCFT exhibit much richer physics, such as the UV/IR mixing [5, 6, 7]. One of the recent additions to the list of intriguing loop physics is the appearance of the generalized $*$-products in the non-planar one-loop amplitudes [8]. In the context of the NCFTs, these generalized $*$-products appear in the one-loop double trace terms in the effective action [8, 9], in certain $U(1)$ anomaly computations [10] and in the expansion of open Wilson line [11, 12]. They also play an essential role in the construction of the solutions of the Seiberg-Witten map and in the issue of the gauge invariance in nonlocal theories [12, 13, 14, 15, 16]. See also [17]. Furthermore, in an apparently unrelated context of the closed string absorption amplitudes at the disk level, the same kind of generalized $*$-products appear [18, 14, 15, 16].

The main theme of the present note is to uniformly understand the appearance of the generalized $*$-products in various contexts by studying the factorization limits of the string amplitudes. This line of approach was first suggested in Ref. [18] for a closed string coupling to two open string states[2]. For example, the fact that both the closed string absorption amplitudes and the

[1] Bearing in mind that a string diagram reduces to a set of field theory diagrams in the decoupling limit, we will use the terms 'tree,' 'one-loop' and so on interchangeably with 'disk,' 'annulus,' etc.

[2] In [19], it was observed that the bulk closed string vertex insertions and the open string loops can be treated on an equal footing in the factorization limit.

one-loop double trace terms in the effective action of Yang-Mills theory contain the same generalized $*$-product structures is natural from our analysis[3]. In this process, we obtain a simple and explicit expression for the $*_N$-products in a closed form for arbitrary $N$ in the non-Abelian context. Furthermore, our analysis indicates that, in the case of the multi-loop amplitudes in the open string context and in the case of the multiple closed string absorption amplitudes on a disk, there might be yet another generalized class of $*$-products.

Our presentation does not sensitively depend on the existence of supersymmetry; when there are supersymmetries, some terms that we write down vanish as the cancellation between the space-time bosonic and fermionic contributions occurs. Each contribution, however, contains the generalized $*$-product parts. In addition, our analysis should be applicable to space-time, space-space and light-like noncommutative field/string theories [20].

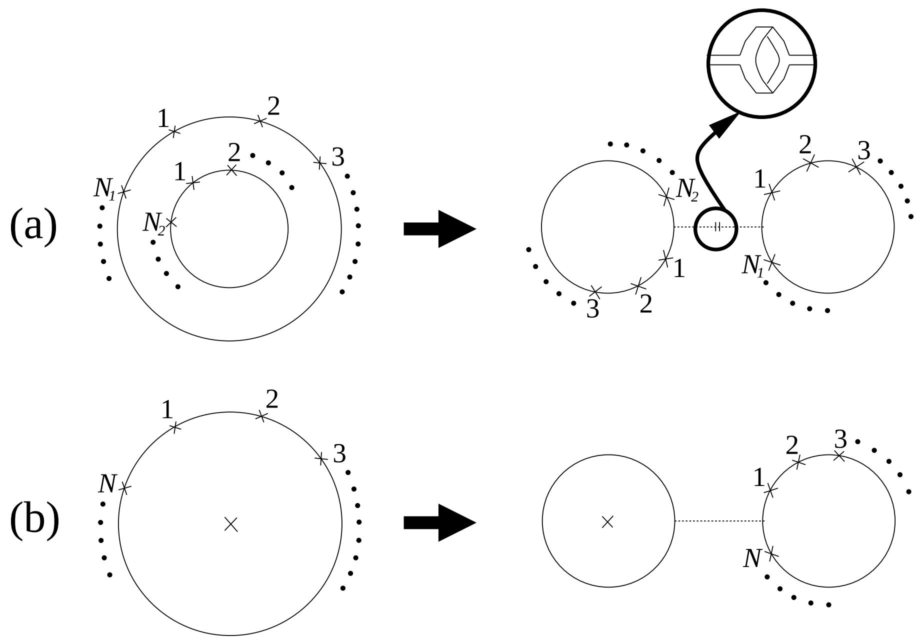


Figure 1: The factorization limits for the open string annulus amplitudes and the disk closed string absorption amplitude. The double twisted line should be used for the connecting leg in the case of annulus amplitude.

The consideration of the factorization limit is helpful in understanding the reason why the coupling of a single closed string with $N$ open strings gives the same $*_N$-product structure as the one-loop amplitude involving purely open string insertions. The $*$-product that appears at the tree level and in the planar multi-loop amplitudes [21] does not depend on the moduli of

[3]Recently, from the analysis of the noncommutative Born-Infeld action, an alternative explanation for the same fact was reported in [15].

the world-sheet and the local position of the open string vertex insertions, but only depends on the ordering of the vertex insertions along a connected boundary. Keeping this in mind, we first consider the open string one-loop amplitude shown in Fig. 1(a).

The case of interest is the factorization limit of the amplitude where the boundary inside approaches the outer boundary. The amplitude in this limit factorizes into the product of two disk amplitudes, each of which hosting the external open string vertex insertions. The connecting leg in this case represents the momentum flowing between two different boundaries through a 'mini-annulus.' As we will find out soon, the object that should be computed to produce the $*_N$-products is the moduli independent part of the disk amplitude with a connecting leg inserted as shown in Fig. 2.

Next, we consider the single closed string absorption amplitude, corresponding to Fig. 1(b). We suppose that enough number of open string vertices are inserted to fix the $SL(2,\mathbb{R})$ invariance of the disk world-sheet. The relevant factorization limit in this case is the limit where the bulk closed string vertex approaches the disk boundary. The amplitude in consideration becomes the product of a disk amplitude with a bulk closed string insertion and another disk amplitude with $N$ boundary insertions. In this factorization limit, no 'mini-annulus' is involved. However, the closed string insertion can be regarded as the specification of the boundary state along the 'blown-up' insertion point. As the incoming closed string momentum passes through the 'cylinder' (which corresponds to the disk on the left hand side of Fig. 1(b)), it becomes topologically similar to the 'mini-annulus.' Noting that the $*_N$-product comes from the moduli-independent part of the amplitudes, which does not change under the change of the moduli, we again see that the relevant object is the disk amplitude with a connecting leg insertion in Fig. 2.

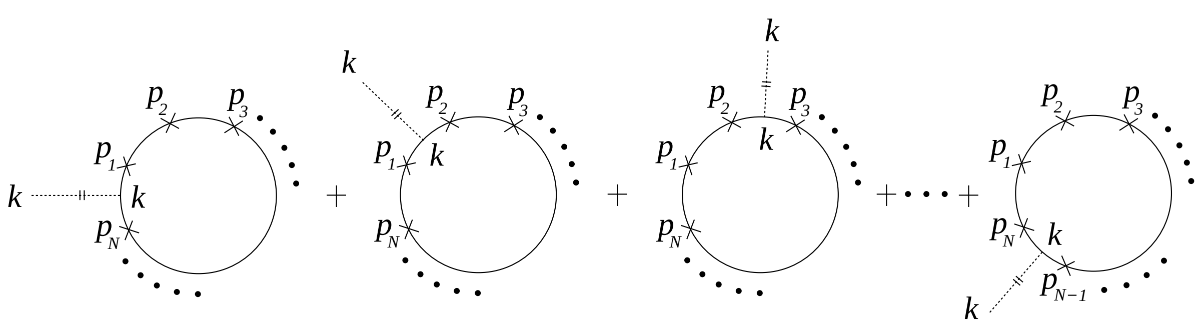


Figure 2: All $N$ cyclic permutations, which have the same Chan-Paton factor, should be added. The first term in the summation in the figure is computed in (8). From the factorization point of view, the summation is automatically generated for a given fixed ordering of the vertices along the disk boundary.

To obtain the explicit expression for the $*_N$-products, we compute the object in Fig. 2 from the open string point of view. As it should, the computation gives the identical answer to the one obtained by the computation of the single closed string absorption amplitude [16]. In Fig. 2, there are $N$ open string vertex insertions along the boundary of the disk. There are $N! = N \times (N-1)!$ permutations of the external vertices; among these $N!$ permutations, the $N$ cyclic permutations do not change the Chan-Paton factor $\mathrm{Tr}(T_1 \cdots T_N)$. These cyclic permutations are summed in Fig. 2. In fact, from the factorization point of view, the summation over the cyclic permutations is automatically generated for a fixed ordering of the vertices along the disk boundary; the connecting leg can be attached to any segment between the two external vertices insertions (corresponding to different factorization channels). On the other hand, non-cyclic permutations change the Chan-Paton factor (assuming that the number of D-branes are large enough) and should be considered independently.

From [21], the one-loop planar and non-planar open string propagators are given by

$$
\begin{aligned}
G_P^{\mu\nu}(z,z') &= \alpha' G^{\mu\nu} G(z,z') + \frac{i}{2}\theta^{\mu\nu}\varepsilon(z-z'), && (4)\\
G_{NP}^{\mu\nu}(z,z') &= \alpha' G^{\mu\nu} G(z,z') - \frac{(\theta G\theta)^{\mu\nu}}{2\pi\alpha' T}(x-x')^2 - \frac{2i}{T}\theta^{\mu\nu}(x-x')(y+y'), && (5)
\end{aligned}
$$

where the function $G(z,z')$ is defined as

$$
G(z,z') = -\log\left|\frac{\theta_1(z-z'|iT)}{\theta_1'(0|iT)}\right|^2 + \frac{2\pi}{T}(y-y')^2 \; . \tag{6}
$$

Here the function $\varepsilon(z-z') = \mathrm{sgn}(y-y')$ for the boundary at $x=0$ and $\varepsilon(z-z') = -\mathrm{sgn}(y-y')$ for the boundary at $x = 1/2$. The momentum $k$ flowing along the connecting leg, which is identical to the momentum flowing between two boundaries, satisfies the momentum conservation condition

$$
k + p_1 + \cdots + p_N = 0 \; . \tag{7}
$$

At the level of the scattering amplitudes ($\exp(-p_\mu G^{\mu\nu} p_\nu)$), there are non-planar contribution coming from the contractions between the free edge point (located at $x=0$ and $y=0$) and the points along the disk (located at $x' = 1/2$ and $y' = \tau_i T$). Furthermore, among the points along the disk, there are planar contributions.

Collecting the terms of the order of $T^0$, which are moduli-independent, from (4) and (5), we have (for the particular ordering of the external vertices as shown in the first term of Fig. 2):

$$
\mathcal{A}_P \mathcal{A}_{NP} = \exp\left[\frac{i}{2}\sum_{i<j} p_i \times p_j\right] \int_0^1 d\tau_1 \int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{N-1}} d\tau_N \exp\left[-i\sum_{i=1}^{N} k \times p_i \; \tau_i\right] \; , \tag{8}
$$

where the first exponential term $\mathcal{A}_P$ originates from the planar contribution, and the second exponential term $\mathcal{A}_{NP}$ comes from the non-planar contribution[4]. When writing down (8), one should keep in mind that

$$\left(\sum_{i=1}^{N} p_i\right) \times k = -k \times k = 0 \tag{9}$$

in view of the momentum conservation condition (7). Noting the identities

$$\sum_{i<j} p_i \times p_j(\tau_i - \tau_j) = \frac{1}{2}\sum_{i,j} p_i \times p_j(\tau_i - \tau_j) = -\sum_{i,j} p_j \times p_i\,\tau_i = \sum_i k \times p_i\,\tau_i \tag{10}$$

we find that (8) is identical to the definition given in [8]. Written in the form of (8), it is obvious that when there is no momentum flow between the two boundaries, $k = 0$, the conventional planar phase term defining the ordinary $*$-product is enough; in particular, this situation includes the case of the planar vertex insertions or, equivalently, the single trace terms in the effective action.

The integration involved in $\mathcal{A}_{NP}$ can be evaluated to yield:

$$\mathcal{A}_{NP}(p_1, \cdots, p_N) = \mathcal{A}^1_{NP}(p_1, \cdots, p_N) + \mathcal{A}^2_{NP}(p_1, \cdots, p_N) \tag{11a}$$

$$\mathcal{A}^1_{NP} = \frac{1}{\prod_{i=2}^{N}(-ik \times P_i)} \tag{11b}$$

$$\mathcal{A}^2_{NP} = \sum_{\epsilon}{}'(-1)^{\sum_{i=2}^{N}\epsilon_i}\frac{1 - \exp(-ik \times q_1^\epsilon)}{\prod_{i=1}^{N}(ik \times q_i^\epsilon)}\,, \tag{11c}$$

The various objects appearing in (11) are as follows. For efficient bookkeeping, we introduced an $(N-1)$ objects $\epsilon = (\epsilon_2, \cdots, \epsilon_N)$ with each $\epsilon_i = 0$ or $1$. The primed summation in (11c) is over all possible values of $\epsilon$ except the case $\epsilon_2 = \cdots = \epsilon_N = 1$, which we separated in (11b). The $N$ objects $q^\epsilon = (q_1^\epsilon, \cdots, q_N^\epsilon)$ is the following linear combination of $p_i$'s;

$$q_i^\epsilon = \sum_{j=i}^{N}\left(\prod_{k=i+1}^{j}\epsilon_k\right)p_j = p_i + \epsilon_{i+1}p_{i+1} + \epsilon_{i+1}\epsilon_{i+2}p_{i+2} + \cdots + \epsilon_{i+1}\epsilon_{i+2}\cdots\epsilon_N p_N\,, \tag{12}$$

In particular, this implies that

$$q_1^\epsilon = p_1 + \epsilon_2 p_2 + \epsilon_2\epsilon_3 p_3 + \cdots + \epsilon_2\epsilon_3\cdots\epsilon_N p_N \quad \text{and} \quad P_i \equiv q_i^{(1,1,\cdots,1)} = \sum_{j=i}^{N} p_i\,. \tag{13}$$

[4]H. Liu informed us that the particular form of $*_N$-products given in (8) is the one that naturally comes from the open Wilson lines. See Eqs. (2.6)-(2.13) of [13].

Remarkably, the complicated $\mathcal{A}_{NP}^2$ term in (11c) completely cancels out when summed over $N$ cyclic permutations. That is,

$$\mathcal{A}_P(p_1, \cdots, p_N)\mathcal{A}_{NP}^2(p_1, \cdots, p_N) + (\text{cyclic permutations}) = 0\ . \tag{14}$$

For the proof, it is convenient to classify the terms in the summation by the value of $|\epsilon| \equiv \sum_{i=1}^{N-1} \epsilon_i$. For each value of $|\epsilon|$ ranging from $0$ to $N-2$ ($|\epsilon| = N-1$ is excluded from the summation), there are ${}_{N-1}C_{|\epsilon|}$ terms; all the terms having the same $|\epsilon|$ have the same sign $(-1)^{|\epsilon|}$ in $\mathcal{A}_{NP}^2$. As for the $e^{-ik\times q_1^\epsilon}$ term in the numerator, note that $q_1^\epsilon = p_1 + \cdots + p_k$ if $k$ is the smallest integer satisfying $\epsilon_2 = \cdots = \epsilon_k = 1$ and $\epsilon_{k+1} = 0$. When multiplied by this exponential factor, the planar phase gets shifted by $k$ steps in cyclic permutation:

$$\begin{aligned} \mathcal{A}_P(p_1, \cdots p_N) \cdot e^{-ik\times(p_1+\cdots+p_k)} &= \mathcal{A}_P(-p_1, \cdots, -p_k, p_{k+1}, \cdots p_N) \\ &= \mathcal{A}_P(p_{k+1}, \cdots, p_N, p_1, \cdots, p_k). \end{aligned} \tag{15}$$

Going to the second line of (15), we use the property that the $k$ step cyclic permutation for the $\mathcal{A}_P$ corresponds to the transformation $p_1, \cdots, p_k \rightarrow -p_1, \cdots, -p_k$. By writing out the denominators and comparing them among different cyclic permutations, one can show that the $e^{-ik\times q_1^\epsilon}\mathcal{A}_P$ terms cancel the $-1 \times \mathcal{A}_P$ terms completely for each value of $|\epsilon|$. Take $N = 3$ and $|\epsilon| = 2$ for example. Up to an overall normalization, the $e^{-ik\times q_1^\epsilon}\mathcal{A}_P$ terms are

$$\begin{array}{ll} (123): & \frac{\mathcal{A}_F(312)}{k\times(p_1+p_2)\ k\times p_2\ k\times p_3} + \frac{\mathcal{A}_F(231)}{k\times p_1\ k\times(p_2+p_3)\ k\times p_3} \\ (231): & \frac{\mathcal{A}_F(123)}{k\times(p_2+p_3)\ k\times p_3\ k\times p_1} + \frac{\mathcal{A}_F(312)}{k\times p_2\ k\times(p_3+p_1)\ k\times p_1} \\ (312): & \frac{\mathcal{A}_F(231)}{k\times(p_3+p_1)\ k\times p_1\ k\times p_2} + \frac{\mathcal{A}_F(123)}{k\times p_3\ k\times(p_1+p_2)\ k\times p_2}, \end{array} \tag{16}$$

while the $(-1)$ terms are

$$\begin{array}{ll} (123): & -\frac{\mathcal{A}_F(123)}{k\times(p_1+p_2)\ k\times p_2\ k\times p_3} - \frac{\mathcal{A}_F(123)}{k\times p_1\ k\times(p_2+p_3)\ k\times p_3} \\ (231): & -\frac{\mathcal{A}_F(231)}{k\times(p_2+p_3)\ k\times p_3\ k\times p_1} - \frac{\mathcal{A}_F(231)}{k\times p_2\ k\times(p_3+p_1)\ k\times p_1} \\ (312): & -\frac{\mathcal{A}_F(312)}{k\times(p_3+p_1)\ k\times p_1\ k\times p_2} - \frac{\mathcal{A}_F(312)}{k\times p_3\ k\times(p_1+p_2)\ k\times p_2}. \end{array} \tag{17}$$

Clearly, every single term in (16) cancels with a term in (17).

Using (14), one can thus write the following simple expression for the $*_N$-products in the momentum space:

$$\begin{aligned} \mathrm{Tr}\ *_N [f_1(p_1), f_2(p_2), \cdots f_N(p_N)] = \sum_{(N-1)!} f_1^{a_1}(p_1) \cdots f_N^{a_N}(p_N) \mathrm{Tr}\,(T^{a_1} \cdots T^{a_N}) \\ \times \left( \frac{\exp\left[\frac{i}{2}\sum_{i<j} p_i \times p_j\right]}{\prod_{i=2}^N (-ik \times P_i)} + (\text{cyclic permutations}) \right) \end{aligned} \tag{18}$$

where the summation runs over the independent Chan-Paton factor, $f_i = \sum_{a_i} f_i^{a_i} T^{a_i}$, $T^{a_i}$ are generators of the Chan-Paton group $U(n)$ for $n$ D-branes, and $k = -(p_1 + \cdots + p_N)$. Written explicitly for $N = 3$, for example, $\mathrm{Tr} *_3 [,,]$ contains

$$\mathrm{Tr}\left(T^{a_1}T^{a_2}T^{a_3}\right)\left(\frac{e^{\frac{i}{2}(p_1\times p_2+p_1\times p_3+p_2\times p_3)}}{-(k\times p_3)(k\times(p_2+p_3))}+(\mathrm{cyclic})\right)+(1\leftrightarrow 2)\,, \tag{19}$$

where $-k = p_1 + p_2 + p_3$, which is identical to Eq. (3.17) of [8]. For $N = 4$, we get

$$\begin{aligned}\mathrm{Tr}\left(T^{a_1}T^{a_2}T^{a_3}T^{a_4}\right) &\left(\frac{e^{\frac{i}{2}(p_1\times p_2+p_1\times p_3+p_1\times p_4+p_2\times p_3+p_2\times p_4+p_3\times p_4)}}{i(k\times p_4)(k\times(p_3+p_4))(k\times(p_2+p_3+p_4))}+(\mathrm{cyclic})\right) \\ &+(5\ \mathrm{noncyclic\ permutations}).\end{aligned} \tag{20}$$

For general value of $N$, there are $N!$ terms in the definition of $*_N$-products. In Ref. [13], a descent relation between $*_N$ and $*_{N+1}$ is given in Eq. (2.18). It is straightforward to show that (18) satisfies the descent relation. Reversing the logic, it might be possible to derive (18) from the descent relation.

Returning to the open string amplitude shown in Fig. 1(a), we find that the effective action from that amplitude contains the $*_N$ products (by computing the two diagrams of the type shown in Fig. 2 and multiplying them)

$$\mathrm{Tr} *_{N_1}\Big[\underbrace{F,\cdots,F}_{N_1}\Big]\mathrm{Tr} *_{N_2}\Big[\underbrace{F,\cdots,F}_{N_2}\Big]\,, \tag{21}$$

when written schematically. By the similar token, the closed string absorption amplitude in Fig. 1(b) yields a term

$$h\mathrm{Tr} *_N\Big[\underbrace{F,\cdots,F}_{N}\Big] \tag{22}$$

among the terms in the effective action, where $h$ is the closed string field. A cautionary remark should be in order[5]. In general, the intermediate modes running through the connecting leg, involved in the factorization, contain infinite number of modes. The schematic factorized forms in (21) and (22) appear for each intermediate mode. When summed over all intermediate modes, however, one generically expect that more complicated (non-factorized) structures arise (see, for example, Eqs. (3.2) and (3.26) of Ref. [8]). In this sense, the terms shown in (21) and (22) are small subsets of terms that can appear in the one-loop effective action, and in the contact terms in the single closed string absorptions on a disk, respectively.

[5]We are grateful to H. Liu and H.-T. Sato for raising this issue.

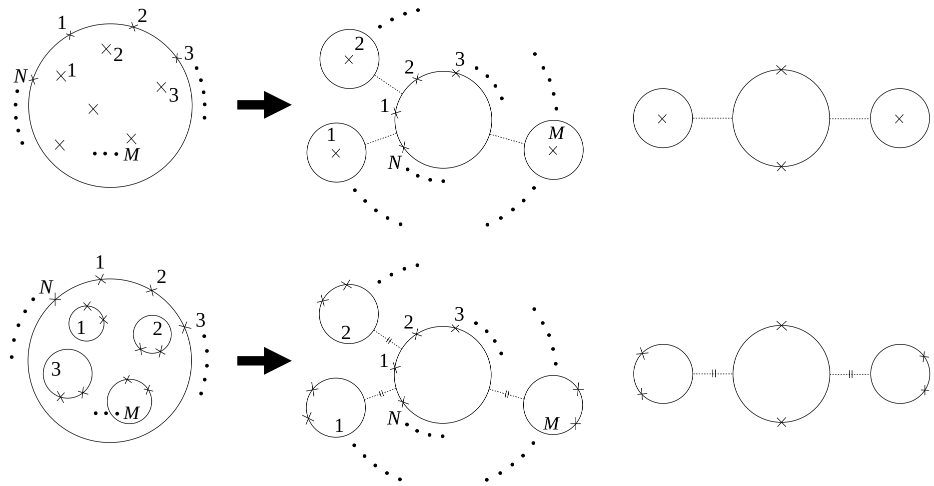


Figure 3: The schematic depiction of the factorizations involving ${}_M{*}_N$-products, which can arise in the disk amplitudes involving $M$ bulk closed string insertions or $M$-loop terms in the open string effective action.

Up to now, our consideration has been restricted to the purely open string insertions at the one-loop level and the tree-level single closed string absorptions. In the $M$-loop ($M > 1$) context for purely open string insertions, in the case of multiple number of closed string absorptions at the tree-level, and in the mixed cases, however, we have more intriguing possibilities. These situations and their interesting factorization limits are depicted in Fig. 3. As illustrated in the middle column of Fig. 3, as many as $M$ connecting legs can be attached to a disk. Generalizing the combinatorics of Fig. 2 to the case of $M$ connecting legs, one now should sum over $(M + N - 1)!$ diagrams, suggesting the possibility of ${}_M{*}_N$ products. Some or all of these products might actually be expressible in terms of $*_N$ products; still, it remains an open question to see if the $*_N$ products can be genuinely further generalized. The simplest nontrivial examples of the ${}_M{*}_N$ products arise from the diagrams in the third column of Fig. 3; the case of two closed string absorptions on a disk and the two-loop triple trace $(\mathrm{Tr}F^2)^3$ terms in the effective action of the supersymmetric Yang-Mills theory. Evaluation of such diagrams and more detailed understanding of ${}_M{*}_N$ products will be reported elsewhere [22].

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